Meson Electro-Magnetic Form Factors in an Extended Nambu–Jona-Lasinio model including Heavy Quark Flavors

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Abstract

Based on an extended NJL model including heavy quark flavors, we calculate the form factors of pseudo-scalar and vector mesons. After taking into account the vector-meson-dominance effect, which introduces a form factor correction to the quark vector coupling vertices, the form factors and electric radii of π^+ and K^+ pseudo-scalar mesons in the light flavor sector fit the experimental data well. The magnetic moments of the light vector mesons ρ^+ and K^{*+} are comparable with other theoretical calculations. The form factors in the light-heavy flavor sector are presented to compare with future experiments or other theoretical calculations.

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I. INTRODUCTION

The Nambu–Jona-Lasinio (NJL) model [1, 2] has been widely used in hadron physics as an effective model to study chiral symmetry in the degree of quark freedom. Usually this model deals with light hadrons composed of only light quark flavors u, d, s with $SU_f(3)$ symmetry [3–6].

In hadron systems including heavy flavors, such as light-heavy mesons, although the chiral symmetry is broken due to the mass of the heavy quark, a complementary heavy flavor symmetry emerges and the so-called heavy quark effective theory (HQET) was formulated for this using the technique of $1/m_Q$ expansion [7–14]. In Ref. [15], the NJL model was extended to include heavy quark flavors to investigate such light-heavy mesons like $D^{(*)}$ and $B^{(*)}$ mesons.

In our previous work [16], we also tried to extend the NJL model to include heavy flavors by expanding the NJL interaction strengths in the inverse power of constituent quark masses according to HQET. Based on this extension, we obtained the meson masses and meson-quark coupling constants of all light and light-heavy mesons in a unified way. Furthermore, the decay widths of the mesons were calculated from those effective meson quark couplings [17].

In this work, we further calculate the electromagnetic form factors of mesons within this extended model. Electromagnetic form factors play an important role in our understanding of hadronic structure. The form factors of the pseudo-scalar mesons π and K were measured in several experiments [18–20] and in some previous theoretical works, the form factors of π and K mesons were studied in the NJL model [21, 22]. After considering the effect of vector-meson-dominance of the vector mesons, such as the ρ meson, in the calculation, typically the form factors of π fit the experimental data well. Furthermore, the form factor of π was also studied in case of finite temperature with the NJL model [23]. Certainly the form factor of π was studied in many other theoretical approaches, such as the Dyson-Schwinger equation using a confining quark propagator [24], light-cone or covariant quark wave functions [25, 26], and the lattice QCD method [27, 28]. Also, with the QCD factorization approach [**? ? ? ?**], the form factor can be extrapolated to higher energy regions by taking into account the perturbative QCD contribution.

The form factors of vector mesons have a rather more complicated structure. Con-

sequently they can provide us more information about vector mesons, such as magnetic moments and quadrupole moments. Presently there are only theoretical results about the form factors of vector mesons. Some works have used the constituent quark model and the light front dynamics [29–31] or Dyson-Schwinger equations [32]. Lattice QCD calculation have been performed with the three-point functions method [33], and the background field method using only two-point functions [34, 35]. The magnetic moments of vector mesons were also calculated by dynamics with the external magnetic field [36], and with QCD sum rules [37].

There are a few papers studying the form factor of light-heavy mesons [38]. These focus on the electroweak form factors. From the heavy flavor symmetry, those form factor should be unifying described by the Isgur-Wise function when the heavy flavor mass goes to infinity.

Here, we perform a systematic calculation of the meson form factors, including pseudo-scalar mesons and vector mesons, of both the light flavor sector and the light-heavy flavor sector, within the extended NJL model. In the next section, we will introduce our model and formalism. The numerical results and discussion will be presented in Section 3.

II. MODEL AND FORMALISM

A. Extended NJL model

To deal with both light and heavy mesons in the Nambu-Jona-Lasinio (NJL) model, in Ref. [16] the four-fermion point interactions are modified to

$$\mathcal{L}_4^F = G_V(\bar{q}\lambda_c^a\gamma^\mu q)(\bar{q}'\lambda_c^a\gamma^\mu q') + \frac{h}{m_q m_{q'}}[(\bar{q}'\lambda_c^a\gamma_\mu q')(\bar{q}\lambda_c^a\gamma_\mu q) + (\bar{q}\gamma_\mu\gamma_5\lambda_c^a q)(\bar{q}'\gamma_\mu\gamma_5\lambda_c^a q')] \quad (1)$$

where λ^a are the generator of $SU(3)$ in color space and $q, q' = u, d, s, c, b$ including both the light and the heavy flavors. Here the second part of the interaction is required to improve the spectra of light vector mesons and the factor of $1/(m_q m_{q'})$ guarantees that the symmetry of heavy flavors will still be hold in the heavy quark limit according to HQET.

By solving the Bethe-Salpeter equation (BSE), we obtain the meson masses and their coupling constants with quarks. We will use the effective Lagrangian to describe the quark

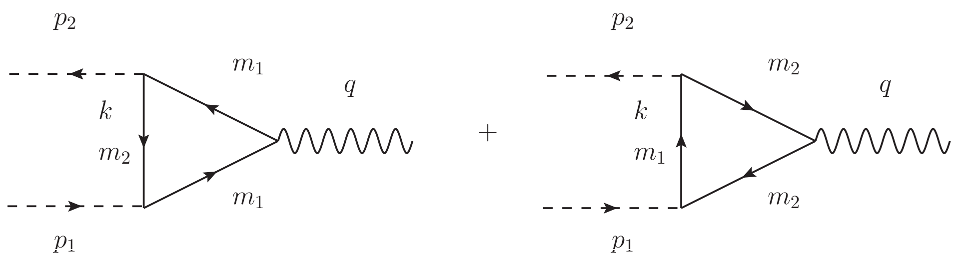

FIG. 1. Feynman diagrams of meson form factor.

interaction in mesons. In the case of π and ρ, the effective Lagrangian reads

$$\mathcal{L}_{\pi qq} = -\, g_{\pi q}\bar{q}i\gamma_5\boldsymbol{\tau}q\cdot\boldsymbol{\pi} - \frac{\tilde{g}_{\pi q}}{2m_q}\bar{q}\gamma_\mu\gamma_5\boldsymbol{\tau}q\cdot\partial^\mu\boldsymbol{\pi}, \tag{2}$$

$$\mathcal{L}_{\rho qq} = -\, g_{\rho q}\bar{q}\gamma_\mu\boldsymbol{\tau}q\cdot\boldsymbol{\rho}. \tag{3}$$

Here the couplings $g_{\pi q}$, $\tilde{g}_{\pi q}$ and $g_{\rho q}$ are treated as constants since the energy of immediate quarks is truncated to the low energy region in the NJL model.

In ref. [17], we have calculated the strong and radiative decays of vector mesons. In this work, we will use the above effective meson Lagrangian to further calculate the form factors of mesons.

B. Form factor of pseudo-scalar mesons

The definition of the form factor of a pseudo-meson is given by

$$\langle\pi^+(p_2)|\bar{\psi}\gamma_\mu\psi|\pi^+(p_1)\rangle = (p_1+p_2)_\mu F(q^2), \tag{4}$$

where $q = p_1 - p_2$ is the transfer momentum. Its Feynman diagrams are shown in Fig. 1 where m_1 and m_2 are the masses of the constituent quarks in the pseudo-scalar meson.

Using the Feynman rules, the amplitude reads

$$(p_1+p_2)_\mu F(q^2) =(p_1+p_2)_\mu[Q_1F^{(1)}(q^2)+Q_2F^{(2)}(q^2)] \tag{5}$$

$$\begin{aligned}(p_1+p_2)_\mu F^{(1)}(q^2) =& -\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\gamma_\mu S_1(k+p_1)i(g-\tilde{g}\frac{\not p_1}{m_1+m_2})i\gamma_5 S_2(k)\\ &\times i(g+\tilde{g}\frac{\not p_2}{m_1+m_2})i\gamma_5 S_1(k+p_2),\end{aligned} \tag{6}$$

$$\begin{aligned}(p_1+p_2)_\mu F^{(2)}(q^2) =& -\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\gamma_\mu S_2(k-p_2)i(g+\tilde{g}\frac{\not p_2}{m_1+m_2})i\gamma_5 S_1(k)\\ &\times i(g-\tilde{g}\frac{\not p_1}{m_1+m_2})i\gamma_5 S_2(k-p_1),\end{aligned} \tag{7}$$

where $F^{(1)}$ and $F^{(2)}$ are the form factors of the quark and anti-quark respectively, Q_i is the electron charge of i-th quark,

$$S_i(p)=\frac{i}{\not p-m_i+i\epsilon} \tag{8}$$

is the propagator of the i-th quark, and g and $\tilde{g}$ are the coupling constants of the pseudoscalar meson obtained in our previous work [16].

In the Breit frame, $p_2^0-p_1^0=0$ and $\boldsymbol{p}_1=-\boldsymbol{p}_2$. We introduce

$$p_1 =p+q/2, \qquad\qquad p_2 =p-q/2, \tag{9}$$

where $p\equiv\frac{1}{2}(p_1+p_2)=(p_1^0,0)$, $q=(0,\boldsymbol{q})$. Taking the direction of the z-axis along momentum $\boldsymbol{p}_1$, we find

$$F^{(1)}(q^2) =in_cn_f\int\frac{d^4k}{(2\pi)^4}\Big[g^2\frac{S_1}{D}-\frac{g\tilde{g}}{m_1+m_2}\frac{S_2}{D}-\frac{g\tilde{g}}{m_1+m_2}\frac{S_3}{D}+\frac{\tilde{g}^2}{(m_1+m_2)^2}\frac{S_4}{D})\Big] \tag{10}$$

$$F^{(2)}(q^2) =F^{(1)}\Big(m_1\leftrightarrow m_2, q\to -q, p\to -p\Big), \tag{11}$$

where

$$S_1 =S_0(m_1,m_1,m_2), \tag{12}$$

$$S_2 =m_1S_0(m_1,(k+p-q/2)^2/m_1,m_2)+m_2S_0(m_1,m_1,k^2/m_2), \tag{13}$$

$$S_3 =m_1S_0((k+p+q/2)^2/m_1,m_1,m_2)+m_2S_0(m_1,m_1,k^2/m_2), \tag{14}$$

$$\begin{aligned}S_4 =&m_1^2S_0((k+p+q/2)^2/m_1,(k+p-q/2)^2/m_1,m_2)+k^2S_0(m_1,m_1,m_2)\\ &+m_1m_2S_0((k+p+q/2)^2/m_1,m_1,k^2/m_2)\\ &+m_1m_2S_0(m_1,(k+p-q/2)^2/m_1,k^2/m_2)\end{aligned} \tag{15}$$

$$D =[(k+p+q/2)^2-m_1^2+i\epsilon)[(k+p-q/2)^2-m_1^2+i\epsilon](k^2-m_2^2+i\epsilon), \tag{16}$$

and

$$
\begin{aligned}
S_0(m_1,m_2,m_3) \equiv & 2[m_3(m_1+m_2) - 2k\cdot(k+p)] \\
& + 2\frac{k\cdot p}{p^2}[m_3(m_1+m_2) - m_1 m_2 + p^2 - k^2 - q^2/4].
\end{aligned} \tag{17}
$$

Note that the denominator D of the integrand is invariant under the transformation $\boldsymbol{k} \to -\boldsymbol{k}$.

The electromagnetic radius will be further obtained from the derivative of the form factor via

$$
r = \left[6\frac{dF}{dq^2}\right]^{1/2}_{q^2=0}. \tag{18}
$$

We have

$$
r = \sqrt{Q_1 r_1^2 + Q_2 r_2^2}, \tag{19}
$$

where

$$
r_i = \left[6\frac{dF^{(i)}}{dq^2}\right]^{1/2}_{q^2=0}, \tag{20}
$$

is the radius of the i-th quark.

C. Form factor of vector mesons

The definition of the form factor of the vector meson reads [39, 40]

$$
\begin{aligned}
\langle\rho^+(p_2,\lambda_2)|\bar\psi\gamma_\mu\psi|\rho^+(p_1,\lambda_1)\rangle = & -\epsilon^*(p_2,\lambda_2)\cdot\epsilon(p_1,\lambda_1)(p_1+p_2)_\mu F_1(q^2) \\
& + [\epsilon_\mu(p_1,\lambda_1) q\cdot\epsilon^*(p_2,\lambda_2) - \epsilon^*_\mu(p_2,\lambda_2) q\cdot\epsilon(p_1,\lambda_1)]F_2(q^2) \\
& + \frac{q\cdot\epsilon^*(p_2,\lambda_2) q\cdot\epsilon(p_1,\lambda_1)}{2m^2}(p_1+p_2)_\mu F_3(q^2),
\end{aligned} \tag{21}
$$

where $\epsilon(p_1)$ and $\epsilon(p_2)$ are the polarization vectors of the initial and the final vector meson respectively. Based on the Feynman diagrams, the LHS of Eq. (21) can be written as

$$
\epsilon_\nu(p_1,\lambda_1)\epsilon^*_\lambda(p_2,\lambda_2) G^{\nu\lambda}_\mu, \tag{22}
$$

where

$$
G^{\nu\lambda}_\mu = Q_1 G^{(1)\nu\lambda}_\mu + Q_2 G^{(2)\nu\lambda}_\mu, \tag{23}
$$

$$
G^{(1)\nu\lambda}_\mu = -\,\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\gamma_\mu S_1(k+p_1) i g_V \gamma^\nu S_2(k) i g_V \gamma^\lambda S_1(k+p_2), \tag{24}
$$

$$
G^{(2)\nu\lambda}_\mu = -\,\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\gamma_\mu S_2(k-p_2) i g_V \gamma^\lambda S_1(k) i g_V \gamma^\nu S_2(k-p_1). \tag{25}
$$

Still working in the Breit frame and taking the z-axis along the momentum $\boldsymbol{p}_1$, the polarization vectors are chosen to be

$$\begin{aligned}\epsilon(p_1,\pm) =&\frac{1}{\sqrt{2}}(0,1,\pm i,0), & \epsilon(p_1,0) =&\frac{1}{m}(p_{1z},0,0,p_{10}),\\ \epsilon(p_2,\pm) =&\frac{1}{\sqrt{2}}(0,1,\mp i,0,) & \epsilon(p_2,0) =&\frac{1}{m}(p_{2z},0,0,p_{20}).\end{aligned} \tag{26}$$

To retrieve F_1, we take the time component in Eq. (21) and find that

$$\begin{aligned}\epsilon_\nu(p_1,\lambda_1)\epsilon^*_\lambda(p_2,\lambda_2)G_0^{\nu\lambda} =&-\epsilon^*(p_2,\lambda_2)\cdot\epsilon(p_1,\lambda_1)(p_1+p_2)_0F_1(q^2)\\ &+\frac{q\cdot\epsilon^*(p_2,\lambda_2)q\cdot\epsilon(p_1,\lambda_1)}{2m^2}(p_1+p_2)_0F_3(q^2).\end{aligned} \tag{27}$$

Then F_1 can be obtained via the transverse polarization

$$\epsilon_\nu(p_1,\pm)\epsilon^*_\lambda(p_2,\pm)G_0^{\nu\lambda}=-\epsilon^*(p_2,\pm)\cdot\epsilon(p_1,\pm)(p_1+p_2)_0F_1(q^2). \tag{28}$$

To retrieve F_2, we take the spatial components in eq. (21) and find that

$$\begin{aligned}\epsilon_\mu(p_1,\lambda_1)\epsilon^*_\nu(p_2,\lambda_2)G_i^{\mu\nu} =&-[\epsilon_i(p_1,\lambda_1)\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*(p_2,\lambda_2)-\epsilon^*_i(p_2,\lambda_2)\boldsymbol{q}\cdot\boldsymbol{\epsilon}(p_1,\lambda_1)]F_2(q^2)\\ =&\{[\boldsymbol{\epsilon}(p_1,\lambda_1)\times\boldsymbol{\epsilon}^*(p_2,\lambda_2)]\times\boldsymbol{q}\}_i\,F_2(q^2).\end{aligned} \tag{29}$$

Still each form factor F_j is a charge weight average of form factors of quark and anti-quark in the vector meson,

$$F_j(q^2)=Q_1F_j^{(1)}(q^2)+Q_2F_j^{(2)}(q^2), \tag{30}$$

and

$$F_j^{(2)}(q^2)=F_j^{(1)}\Big(m_1\leftrightarrow m_2, q\to -q, p\to -p\Big).$$

Explicitly we obtain

$$\begin{aligned}F_1^{(1)}(q^2) =&\epsilon_\nu(p_1,+)\epsilon^*_\lambda(p_2,-)G_0^{(1)\nu\lambda}/(2p_0)\\ =&in_cn_fg_V^2\int\frac{d^4k}{(2\pi)^4}\frac{G_1}{D},\end{aligned} \tag{31}$$

where

$$\begin{aligned}G_1 =&4[(k+p)\cdot k-m_1m_2+k_x^2+k_y^2]\\ &-2\frac{p\cdot k}{p^2}[p^2-k^2-q^2/4-m_1^2+2m_1m_2-2(k_x^2+k_y^2)],\end{aligned} \tag{32}$$

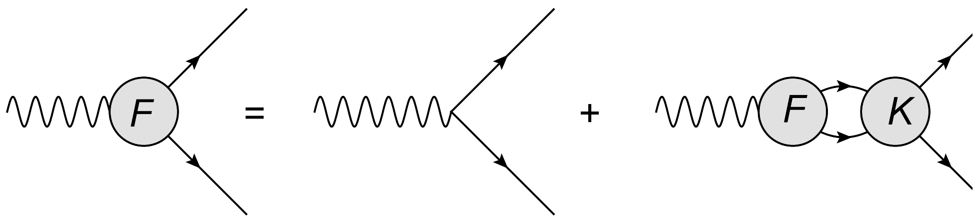

FIG. 2. Feynmann diagram of the loop correction

and

$$
\begin{aligned}
F_2^{(1)}(q^2) =& -\frac{m_V}{p_0|\boldsymbol{q}|}\left[\frac{1-i}{\sqrt{2}}\epsilon_\mu(p_1,+)\epsilon_\nu^*(p_2,0)G_1^{(1)\mu\nu}+\frac{1+i}{\sqrt{2}}\epsilon_\mu(p_1,-)\epsilon_\nu^*(p_2,0)G_2^{(1)\mu\nu}\right] \\
=& i n_c n_f g_V^2\int\frac{d^4k}{(2\pi)^4}\frac{G_2}{D},
\end{aligned}
\tag{33}
$$

where

$$
\begin{aligned}
G_2 =& 4[k\cdot(k+p)-m_1m_2+k_z^2] \\
&-2\frac{k\cdot p}{p^2}[(k+p)^2-q^2/4-m_1^2+2(k_x^2+k_y^2)].
\end{aligned}
\tag{34}
$$

We will not consider the form factor F_3 in this work.

D. Vector-Meson-Dominance and Quark Loop Correction

According to the vector-meson-dominance picture, the π and K form factor are dominated by the ρ, ω and ϕ intermediate vector meson states [21]. In the NJL model, the vector-meson-dominance is represented by the correction to quark-photon vertex as shown in the Feynman diagram in Fig. 2. The correction will introduce a form factor to the constituent quark [5]. For the i-th quark

$$
F_q^{(i)}(q^2) = \frac{1}{1-K^V J_{VV}^{(T)}},
\tag{35}
$$

where K^V is the NJL vector coupling constant and $J_{VV}^{(T)}$ represents the transverse vector loop integral [4, 16]. The meson form factor will be modified to

$$
F(q^2) = Q_1F^{(1)}(q^2)F_q^{(1)}(q^2)+Q_2F^{(2)}(q^2)F_q^{(2)}(q^2).
\tag{36}
$$

TABLE I. The masses and quark coupling constants of pseudo-scalar mesons

	π	K	D	D_s	B
mass(MeV)	139	496	1870	1940	5280
g	4.25	4.32	4.71	5.03	5.92
$\tilde{g}$	1.56	1.61	2.04	2.09	2.84

TABLE II. The masses and quark coupling constants of vector mesons

	ρ	K^*	D^*	D_s^*	B^*
mass(MeV)	771	918	1990	2120	5310
g	1.29	1.31	1.64	1.83	2.51

III. NUMERICAL RESULTS

The parameters of the extended NJL model were fixed by fitting the meson mass spectra and decay constants in a previous work [16]. The input parameters were the current masses of light quarks and the constituent masses of heavy quarks, two coupling constants and the 3-dimensional cutoff:

$$\begin{aligned}
&m_{u/d}^0 = 2.79\text{MeV}, \qquad && m_s^0 = 72.0\text{MeV}, \\
&m_c = 1.62\text{GeV}, && m_b = 4.94\text{GeV}, \\
&\Lambda = 0.8\text{GeV}, && G_V = 2.41, \\
&h = 0.65. &&
\end{aligned} \tag{37}$$

Due to charge conservation, the form factor should be normalized to $F(q^2 = 0) = 1$ for any hadron carrying +1 charge. We will make a self-consistent calculation by using the theoretical values of meson masses and the quark coupling constants together. This will guarantee the strict normalization of the form factor at $q^2 = 0$ [23]. The theoretical values of pseudo-scalar and vector mesons are listed in Table I and Table II respectively.

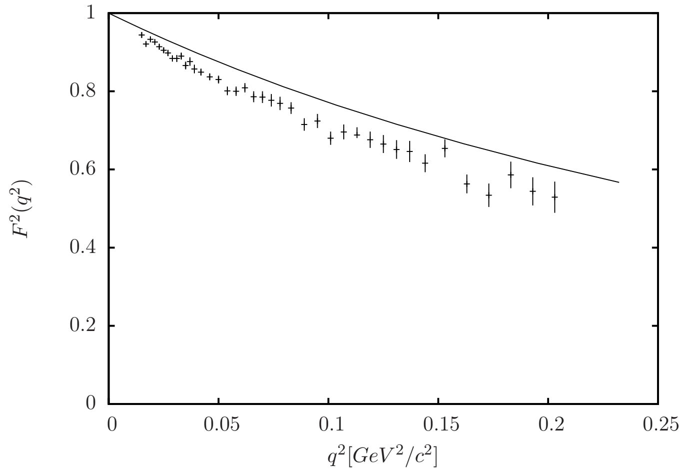

FIG. 3. The form factor of π^+ compared to the experimental data from Ref. [20].

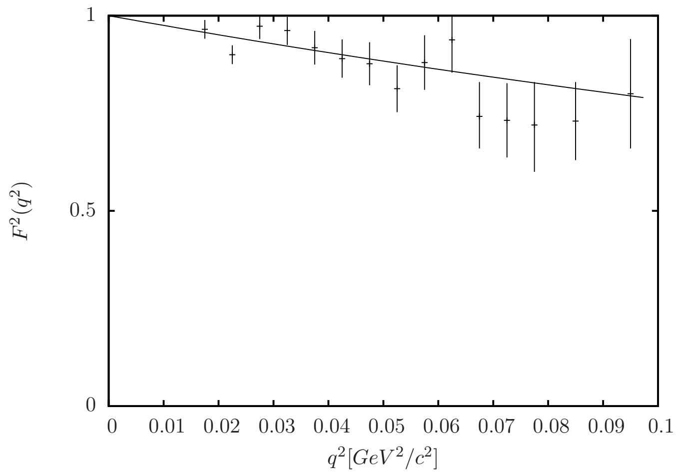

FIG. 4. The form factor of K^+ compared to the experimental data from Ref. [19].

A. Pseudo-scalar mesons

The form factors of π^+ and K^+ are compared with experimental data in Fig. 3 and Fig. 4 respectively. The theoretical results fit the experimental data well. In the theoretical calculation, the quark loop correction is included to account for the important effect of vector-meson-dominance.

The heavy-light pseudo-scalar mesons like D^+, D_s^+ and B^+ have no experimental data for form factor yet. In Fig. 5, we present the form factors of all positive pseudo-scalar mesons.

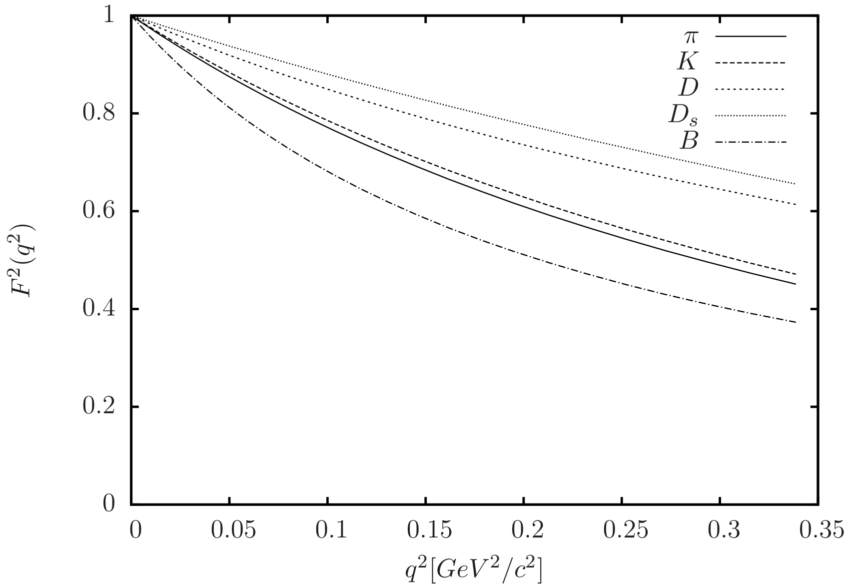

FIG. 5. The form factors of all positive pseudo-scalar mesons.

TABLE III. Comparison of electric radii $\langle r^2\rangle^{1/2}$ of pseudo-scalar mesons vs. experimental data. r_1 and r_2 are constituent quark and anti-quark radii in the meson.

Meson	$\langle r_1^2\rangle$ (fm^2)	$\langle r_2^2\rangle$ (fm^2)	$\langle r^2\rangle^{1/2}_{\text{cal.}}$ (fm)	$\langle r^2\rangle^{1/2}_{\text{exp.}}$ (fm) [19, 20]
π^+	0.322	0.322	0.57	0.66
K^+	0.341	0.206	0.54	0.56
D^+	0.080	0.473	0.46	
D_s^+	0.083	0.283	0.39	
B^+	0.795	0.041	0.74	

The form factor at low momentum q^2 can be well illustrated by the electromagnetic radius. The radii of all positive pseudo-scalar mesons are listed in Table III.

As had been observed in Ref. [21], it is the quark loop correction of vector-meson-dominance that makes the π radius bigger than that of the K. In Eq. (19), the radius of a meson is a charge weight average of individual quark radii. From Eq. (36), after considering the quark loop correction, we have

$$\langle r_i^2\rangle = \langle r_i^2\rangle_{\text{int}} + \langle r_i^2\rangle_q\,, \tag{38}$$

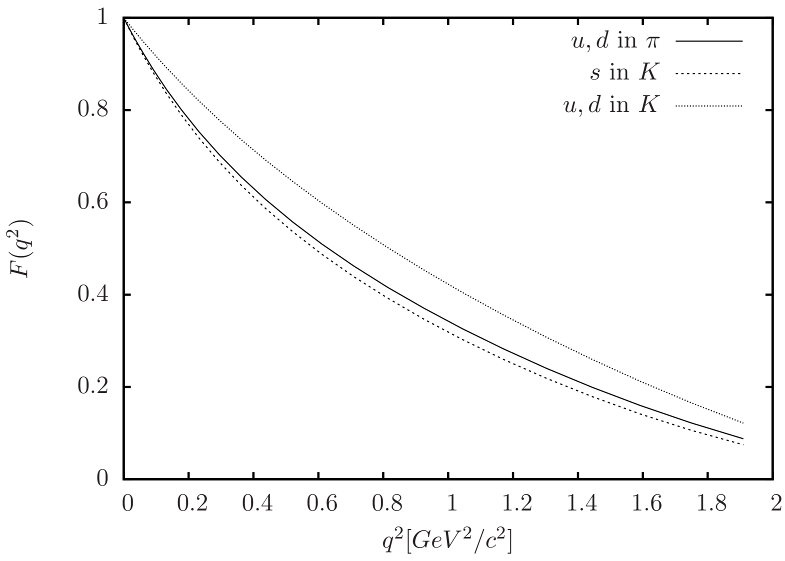

FIG. 6. The form factors of constituent quark and anti-quark in π and K.

where

$$\langle r_i^2 \rangle_{\text{int}} = \left[6 \frac{dF^{(i)}}{dq^2} \right]_{q^2=0}, \tag{39}$$

$$\langle r_i^2 \rangle_q = \left[6 \frac{dF_q^{(i)}}{dq^2} \right]_{q^2=0}, \tag{40}$$

are the "intrinsic" charge radius and the quark loop correction respectively. The quark loop correction decreases as the quark mass increases, so the lighter quark has a larger radius than its heavier partner in any meson. We show the individual form factors of quark and anti-quark in π and K mesons in Fig. 6 and also list the individual quark radii in Table III.

Just like the π^+ and K^+, the radii of the light-heavy mesons $\langle r^2_{D^+} \rangle^{1/2}$, $\langle r^2_{D_s^+} \rangle^{1/2}$ decrease as the meson mass increases. However the radius of the B^+ meson increases by roughly a factor 2. This mainly because, in a light-heavy meson, the heavy quark's contribution is much smaller than that of the light one. If we neglect the contribution of the heavy quark, in the B^+ meson, the u-quark has a 2/3 charge weight of contribution. The d-quark, on the other hand, has only a 1/3 charge weight in the D^+ and D_s^+ mesons. The form factors of individual constituent quarks in D and B are shown in Fig. 7.

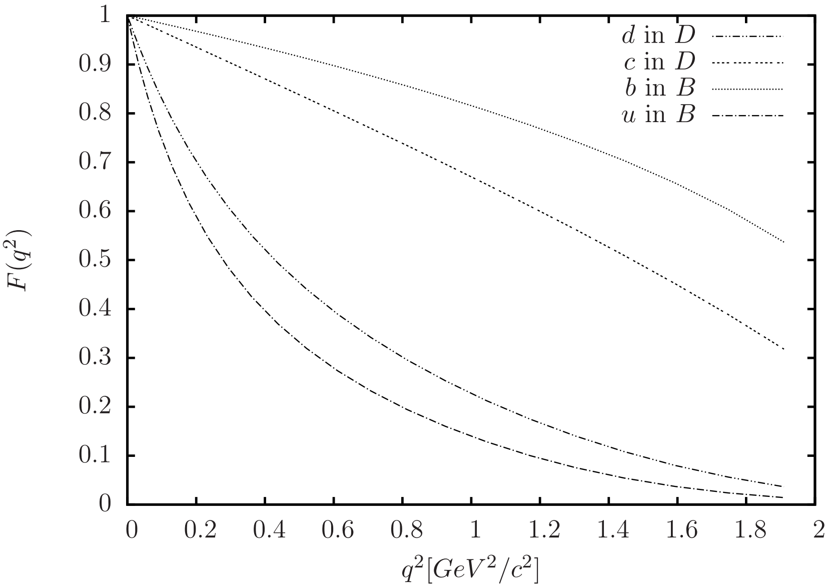

FIG. 7. The form factors of constituent quark and anti-quark in B and D.

B. Vector mesons

The electric form factors F_1 of vector mesons are shown in Fig. 8. The electric radii are listed in Table IV. Because all vector meson masses are close to their thresholds, their bound energies are small and their radii are larger than their pseudo-scalar partners.

The magnetic form factors F_2 are presented in Fig. 9. They are connected to the magnetic momentum through [34]

$$\mu_V = \frac{F_2(0)e}{2m_V}. \tag{41}$$

The magnetic moments are also listed in Table IV. The magnetic moments are given in the unit of nuclear magneton μ_n. Generally, the magnetic momentum decreases as the meson mass increases. In our results, the magnetic moments of D^* and D_s^* are smaller than those of the light mesons ρ and K. However, the magnetic moment of B^* is larger than that of D^* and D_s^*. The reason is still that the main contribution comes from the light quark but the charge of the u is larger than that of the d and s by a factor of 2. Up to now, no experimental data is available. We compare our results for the ρ^+ and K^{*+} mesons with other theoretical work [29] and [33].

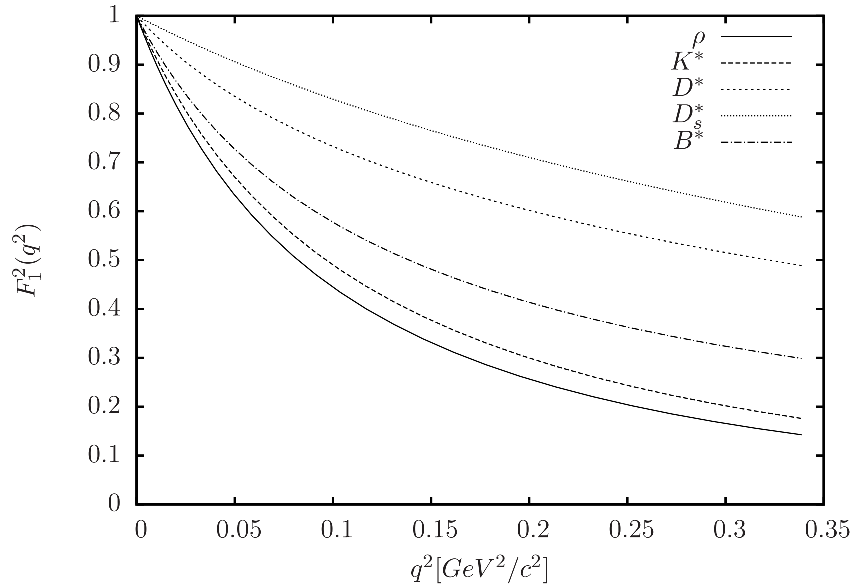

FIG. 8. F_1 of all positive vector mesons with respect to q^2

TABLE IV. Radii and magnetic moments of vector mesons

meson	$r_1^2(fm^2)$	$r_2^2(fm^2)$	$r_c(fm)$	$\mu_1(\mu_n)$	$\mu_2(\mu_n)$	$\mu(\mu_n)$	$\mu(\mu_n)$[29]	$\mu(\mu_n)$[33]
ρ^+	1.267	1.267	1.12	1.69	0.85	2.54	2.56	3.25
K^{*+}	1.304	0.697	1.05	1.63	0.63	2.26		2.81
D^{*+}	0.095	1.366	0.72	0.42	0.74	1.16		
D_s^{*+}	0.083	0.567	0.49	0.42	0.56	0.98		
B^{*+}	1.359	0.025	0.96	1.4	0.07	1.47		

IV. SUMMARY

With the extended NJL model including heavy flavors, we have made a systematic calculation of the form factors of mesons, including the pseudo-scalar mesons and their vector partners, of both the light flavor sector and the light-heavy flavor sector. The form factors of the π and K mesons fit the experimental data. Other form factors of mesons, especially of the light-heavy mesons, are presented here to compare with future experiments and other theoretical calculations such as lattice calculation.

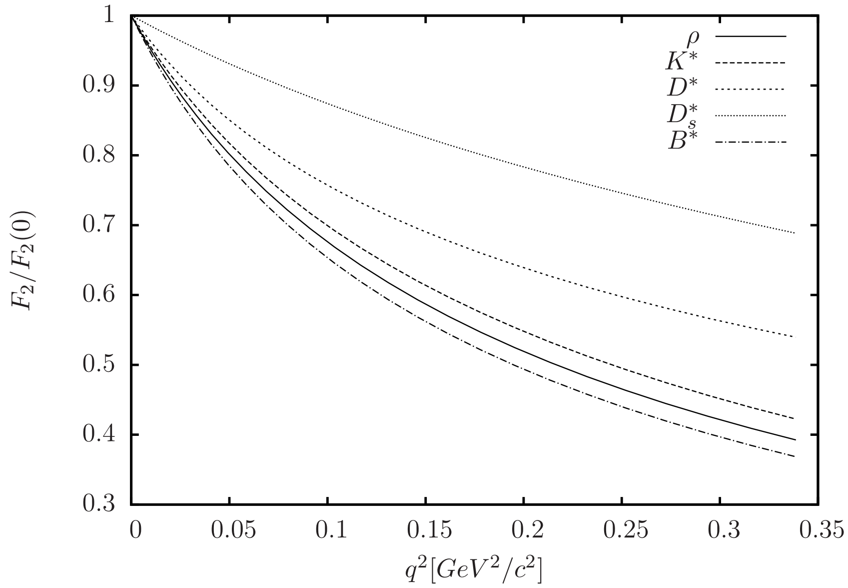

FIG. 9. F_2 of all positive vector mesons with respect to q^2

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[1] Y. Nambu and G. Jona-Lasinio, Phys.Rev. **122**, 345 (1961).

[2] Y. Nambu and G. Jona-Lasinio, Phys.Rev. **124**, 246 (1961).

[3] S. Klevansky, Rev.Mod.Phys. **64**, 649 (1992).

[4] S. Klimt, M. F. Lutz, U. Vogl, and W. Weise, Nucl.Phys. **A516**, 429 (1990).

[5] U. Vogl, M. F. Lutz, S. Klimt, and W. Weise, Nucl.Phys. **A516**, 469 (1990).

[6] U. Vogl and W. Weise, Prog.Part.Nucl.Phys. **27**, 195 (1991).

[7] M. A. Shifman and M. Voloshin, Sov.J.Nucl.Phys. **45**, 292 (1987).

[8] H. D. Politzer and M. B. Wise, Phys.Lett. **B208**, 504 (1988).

[9] N. Isgur and M. B. Wise, Phys.Lett. **B232**, 113 (1989).

[10] E. Eichten and B. R. Hill, Phys.Lett. **B234**, 511 (1990).

[11] H. Georgi, Phys.Lett. **B240**, 447 (1990).

[12] B. Grinstein, Nucl.Phys. **B339**, 253 (1990).

[13] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl.Phys. **B343**, 1 (1990).

[14] T. Mannel, W. Roberts, and Z. Ryzak, Nucl.Phys. **B368**, 204 (1992).

[15] D. Ebert, T. Feldmann, R. Friedrich, and H. Reinhardt, Nucl.Phys. **B434**, 619 (1995), arXiv:hep-ph/9406220 [hep-ph].

[16] X.-Y. Guo, X.-L. Chen, and W.-Z. Deng, Chin.Phys. **C37**, 033102 (2013), arXiv:1205.0355 [hep-ph].

[17] H.-B. Deng, X.-L. Chen, and W.-Z. Deng, Chin.Phys. **C38**, 013103 (2014), arXiv:1304.5279 [hep-ph].

[18] S. Amendolia, B. Badelek, G. Batignani, G. Beck, F. Bedeschi, *et al.*, Phys.Lett. **B146**, 116 (1984).

[19] S. Amendolia, G. Batignani, G. Beck, E. Bellamy, E. Bertolucci, *et al.*, Phys.Lett. **B178**, 435 (1986).

[20] S. Amendolia *et al.* (NA7 Collaboration), Nucl.Phys. **B277**, 168 (1986).

[21] V. Bernard and U. G. Meissner, Phys.Rev.Lett. **61**, 2296 (1988).

[22] M. F. Lutz and W. Weise, Nucl.Phys. **A518**, 156 (1990).

[23] H. Schulze, J.Phys. **G20**, 531 (1994).

[24] C. D. Roberts, Nucl.Phys. **A605**, 475 (1996), arXiv:hep-ph/9408233 [hep-ph].

[25] P. Chung, F. Coester, and W. Polyzou, Phys.Lett. **B205**, 545 (1988).

[26] H. Ito, W. Buck, and F. Gross, Phys.Lett. **B248**, 28 (1990).

[27] R. Woloshyn and A. Kobos, Phys.Rev. **D33**, 222 (1986).

[28] Y. Nemoto (RBC Collaboration), Nucl.Phys.Proc.Suppl. **129**, 299 (2004), arXiv:hep-lat/0309173 [hep-lat].

[29] B. L. Bakker, H.-M. Choi, and C.-R. Ji, Phys.Rev. **D65**, 116001 (2002), arXiv:hep-ph/0202217 [hep-ph].

[30] F. Cardarelli, I. Grach, I. Narodetsky, G. Salme, and S. Simula, Phys.Lett. **B349**, 393 (1995), arXiv:hep-ph/9502360 [hep-ph].

[31] J. de Melo and T. Frederico, Phys.Rev. **C55**, 2043 (1997), arXiv:nucl-th/9706032 [nucl-th].

[32] M. Bhagwat and P. Maris, Phys.Rev. **C77**, 025203 (2008), arXiv:nucl-th/0612069 [nucl-th].

[33] F. X. Lee, S. Moerschbacher, and W. Wilcox, Phys.Rev. **D78**, 094502 (2008), arXiv:0807.4150 [hep-lat].

[34] J. Hedditch, W. Kamleh, B. Lasscock, D. Leinweber, A. Williams, *et al.*, Phys.Rev. **D75**, 094504 (2007), arXiv:hep-lat/0703014 [HEP-LAT].

[35] H. Rubinstein, S. Solomon, and T. Wittlich, Nucl.Phys. **B457**, 577 (1995), arXiv:hep-lat/9501001 [hep-lat].

[36] A. Badalian and Y. Simonov, Phys.Rev. **D87**, 074012 (2013), arXiv:1211.4349 [hep-ph].

[37] T. Aliev, A. Ozpineci, and M. Savci, Phys.Lett. **B678**, 470 (2009), arXiv:0902.4627 [hep-ph].

[38] M. Gomez-Rocha and W. Schweiger, Phys.Rev. **D86**, 053010 (2012), arXiv:1206.1257 [hep-ph].

[39] S. J. Brodsky and J. R. Hiller, Phys.Rev. **D46**, 2141 (1992).

[40] R. Arnold, C. E. Carlson, and F. Gross, Phys.Rev. **C23**, 363 (1981).